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ON THE DYNAMICS OF AN ARMATURE CONTROLLED DC MOTOR MOUNTED ON AN ELASTICALLY SUPPORTED TABLE

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Abstract. *This paper presents some analysis on the dynamics of an unbalanced direct current motor mounted on an elastically supported table through numerical simulations. The governing equations of motion are obtained from the Lagrange's equations. We consider that a non-ideal energy source supplies the energy to the system, and analyze the interactions between dynamics of the foundation and dynamics of the motor. Several authors, as one may see in the paper, worked with non-ideal problems and, particularly, with unbalanced motors mounted on elastic foundations. However, the works presented up until now in the literature don't take into account the "nature of the regulation" of the motor, but only experimental curves for the driving and resisting torque. In this work we consider the "nature of the regulation" of the motor. Hence the problem has one more degree of freedom than the known works. In this way it's possible to take directly into account several parameters intrinsically related to the construction of the motor, not only the characteristics of the foundation.*

Keywords: *Non-ideal systems, Nonlinear systems, Sommerfeld effect, Nonlinear vibrations*

1. INTRODUCTION

The study of the interactions between the several parts that compose a given nonlinear system is important to understand several phenomena observed in practical situations. In this work we are concerned with the interactions between the dynamics of a DC motor and the dynamics of its foundation. This problem can be included in the study of non-ideal systems because the primary source of energy to excite the structure (foundation) is the kinetic energy of the motor. On the other hand, the dynamics of the motor can be altered by the dynamics of the foundation. Furthermore, the most of the electrical DC motors don't have a non-limited power supplier. Hence, we have the case in which the excitation is influenced by the response of the structure, that is, this system is non-ideal. The first kind of non-ideal problem to arise in the literature is the so-called Sommerfeld effect (Kononenko, 1969). Evan-Iwanowski (1976) and Dimentberg *et al* (1994) gave further contributions to this non-ideal problem. Nayfeh & Mook (1979) present a comprehensive and complete review of different approaches until 1979. Recently, Balthazar *et al* (1996-a) analyzed, with interesting details, the governing equations of a non-ideal problem, containing quadratic and cubic nonlinearities, for a cantilevered beam supporting a non-ideal energy source at its free end. Balthazar *et al* (1996-b), Balthazar *et al* (1996-c) and Wieczorek & Mook (1997) present some experimental results on this problem. The numerical simulation of a simplified dynamics of this problem, using an unbalanced rotor attached to an elastic nonlinear support with internal and external damping and driven by a non-ideal energy source were analyzed by Balthazar *et al* (1996-d). The referred works use experimental data to estimate the characteristic curves of the motor. Hence the torque generated by the motor is not known explicitly, but only implicitly, through curves torque versus rotation speed of the motor. However, this curve is generally not obtained when the interactions between the motor and the foundation take place. In this work we take into account the equations that govern the response of the motor and, then, make a coupling between the mechanical equations, that is, the equations that govern the response of the foundation, and the electrical ones. Taking explicitly in account the equations of the motor has some advantages:

(a) The characteristic curve of the motor is easily included in the model of the problem.
(b) It's possible to preview the mechanical response of the system and the electrical one.
(c) If the motor have any kind of control, it's possible to consider it in the model easily.

The next sections presents some analyses on the proposed problem.

2. PHYSICAL SYSTEM AND GOVERNING EQUATIONS

Figure 01 shows a simplified sketch of the system, which consists of elastically supported table (foundation) and an unbalanced motor. The physical parameters indicated in that figure represent:

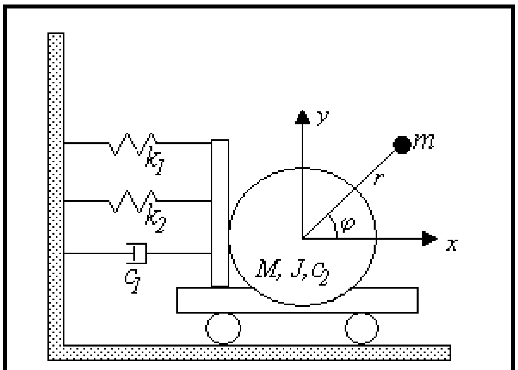

Fig 01 - Physical System

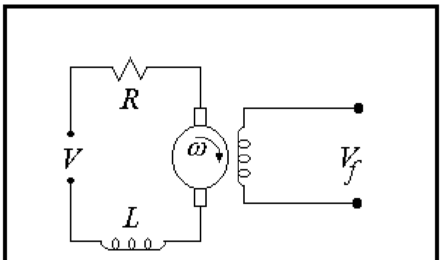

Fig 02 - DC motor Scheme

- k_1 and k_2 : linear and nonlinear stiffness elements, respectively.
- z and θ : translational and rotational coordinates of the displacements of the motor.
- M, J and m: mass of the table + motor set, moment of inertia of the rotor and unbalancing mass, respectively.
- c_1 and c_2 : damping acting at the translational and rotational coordinates, respectively.
- r : distance between the unbalancing mass and the shaft of the motor

In order to obtain the governing equations of motion of the system we used the Lagrange's equations, which leads to the two following equations:

$$\left(M+m\right)\frac{d^2z}{dt^2}+c\frac{dz}{dt}+k_Lz+k_{NL}z^3-mr\frac{d^2\theta}{dt^2}\sin\theta-mr\left(\frac{d\theta}{dt}\right)^2cos\theta=0. \qquad (01)$$

$$\left(J+mr^2\right)\frac{d^2\theta}{dt^2}+c_\theta\frac{d\theta}{dt}-mr\frac{d^2z}{dt^2}\sin\theta+mgrcos\theta=k_tI_a-T\left(t\right). \qquad (02)$$

where k_tI_a is the torque generated by the motor and $T\left(t\right)$ is the resisting torque of the load acting over the motor. Without loss of generality, we can neglect the resisting torque, so that, from now on, we will not consider this torque.

Figure 02 shows the scheme of an armature controlled DC motor. In this case, the electric current in the armature circuit has the following governing equation:

$$L\frac{dI_a}{dt}+R_aI_a+e_b=L\frac{dI_a}{dt}+R_aI_a+k_b\frac{d\theta}{dt}=V\left(t\right). \qquad (03)$$

where e_b is the counterelectromotive force. The torque generated by the motor is written as

$$T_1=k_tI_a. \qquad (04)$$

To obtain the governing equations of the system in their dimensionless form we define the non-dimensional time as $\tau=\omega_0t$, the non-dimensional displacement as $x=\frac{z}{r}$ and:

$$\left\{\begin{array}{ccc}\omega_0=\sqrt{\dfrac{k_1}{M+m}} & \dfrac{c_1}{M+m}=2\xi\omega_0 & \dfrac{c_2}{J+mr^2}=2\xi_\theta\omega_0\\ v=\dfrac{k_t}{Rc_2+k_bk_t}V & i=\dfrac{k_t}{c_2\omega_0}I_a & \hat{T}=\dfrac{T(t)}{\left(J+mr^2\right)\omega_0^2}\\ \dfrac{m}{M+m}=\mu & \dfrac{mr^2}{J+mr^2}=\varepsilon & \dfrac{R_a}{L_a\omega_0}=\rho\\ \dfrac{g}{\omega_0^2r}=\kappa & \dfrac{k_2r^2}{k_1}=\sigma & \dfrac{k_bk_t}{R_ac_2}=\lambda\end{array}\right\}. \qquad (05)$$

Hence we obtain

$$\frac{d^2x}{d\tau^2}+2\varsigma\frac{dx}{d\tau}+x+\sigma x^3-\mu\left[\frac{d^2\theta}{d\tau^2}\sin\theta-\left(\frac{d\theta}{d\tau}\right)^2cos\theta\right]=0. \qquad (06)$$

$$\frac{d^2\theta}{d\tau^2}+2\varsigma_\theta\frac{d\theta}{d\tau}-\varepsilon\left[\frac{d^2x}{d\tau^2}\sin\theta-\kappa\cos\theta\right]=2\varsigma_\theta i-\hat{T}. \qquad (07)$$

$$\frac{di_a}{d\tau} + \rho\, i_a + \rho \cdot \lambda \ \frac{d\theta}{d\tau} = \rho\,(1+\lambda)v\,. \tag{08}$$

To integrate the equations (06) to (08) in a safer way we should manipulate them. Note that both equations (06) and (07) have explicitly the second derivative of x and φ. This generates an algebraic loop during the integration of the equations and the results can be expurious. After some manipulations we have

$$\begin{aligned} &\left(1-\mu\,\varepsilon\,\sin^2\theta\right)\frac{d^2x}{d\tau^2} + 2\varsigma\ \frac{dx}{d\tau} + x + \sigma x^3 + 2\varsigma_\theta\mu\ \frac{d\theta}{d\tau}\sin\theta + \frac{\mu\,\varepsilon\kappa}{2}\ \sin 2\theta \\ &-2\mu\varsigma_\theta\ i\sin\theta + \mu\, m\sin\theta + \mu\left(\frac{d\theta}{d\tau}\right)^2\cos\theta\ = 0 \end{aligned} \quad . \tag{09}$$

$$\begin{aligned} &\left(1-\mu\,\varepsilon\ \sin^2\theta\right)\frac{d^2\theta}{d\tau^2} + 2\varsigma_\theta\ \frac{d\theta}{d\tau} + 2\varsigma\,\varepsilon\ \frac{dx}{d\tau}\sin\theta\ + \varepsilon\ x\sin\theta\ - \varepsilon\,\sigma\ x^3\sin\theta\ - \\ &-\frac{\varepsilon\ \mu}{2}\left(\frac{d\theta}{d\tau}\right)^2\sin 2\theta\ + \varepsilon\,\kappa\ \cos\theta\ - 2\varsigma_\theta\ i + m = 0 \end{aligned} \quad . \tag{10}$$

$$\frac{di}{d\tau} + \rho\, i + \rho \cdot \lambda \ \frac{d\theta}{d\tau} = \rho\,(1+\lambda)v\,. \tag{11}$$

Now it's easy to see that the coefficients of the highest derivative terms are not constant. Note that these coefficients could reach to zero if the product $\mu\,\varepsilon$ reaches to the unity. However, it's not possible in practice because both μ and ε are less than the unity.

Equations (09) to (11) were analyzed through numerical simulations. Note that the governing equations take in account the gravity field and the mechanical and electrical coupling, which can be important to explain some phenomena observed in practical situations.

3. A NOTE ON EXPERIMENTAL OBSERVATIONS

Some experimental observations on the dynamics of an unbalanced motor attached to a beam were carried out at Universidade Estadual de Campinas (Campinas-SP-Brazil), Universidade Federal do Espírito Santo (Vitória-ES-Brazil) and Virginia Polytechnic Institute and State University (Blacksburg-VI-USA). A detailed description of the experimental setups used in the tests can be found in (Mattos et al, 1997-a) and (Mook et al, 1997-b). The experimental setup used in the tests performed in Brazil consists of a cantilevered beam with an unbalanced DC motor at its free end. The response of the beam was monitored by a noncontacting inductance gage (Unicamp) and by an accelerometer near the end of the beam (Vitória). To control the speed of the motor a power supplier was used and the operator controlled the applied voltage. The current was left free according to the demand of the motor, but the available power was limited. The response of the beam was monitored by a chain-driven encoder with 560 pulses per revolution.

Figure 03 shows the rotation speed of the motor as a function of applied voltage in a test performed in Vitória. Note that the rotation speed of the motor doesn't change for voltage between 6 and 9 volts for increasing voltage. At 9 volts, the rotation speed jumps from 7.5 Hz to 15.75 Hz. The value of 7.5 Hz is very closed to the natural frequency of the beam. For decreasing voltage the jump is much smaller and occurs at 5.9 volts, when the rotation speed jumps from 8.375 Hz to 7.5 Hz. The amplitude of vibration of the beam as a function of the applied voltage is shown in figure 04, where one can note the jump in the amplitude of the system response at 9 volts, in the opposite direction of the rotation speed of the motor.

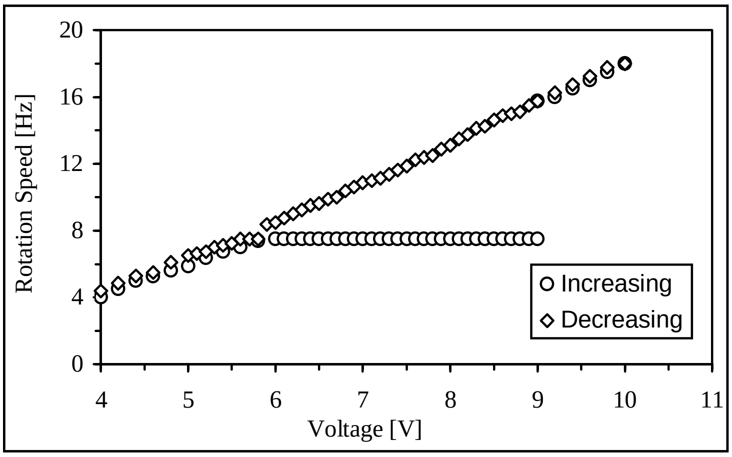

Fig 03 - Rotation speed of the motor vs. applied voltage

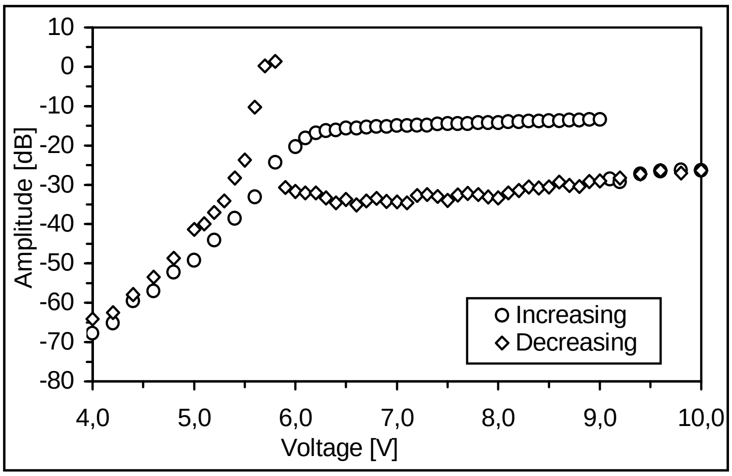

Fig 04 - Amplitude of first harmonic vs. applied voltage

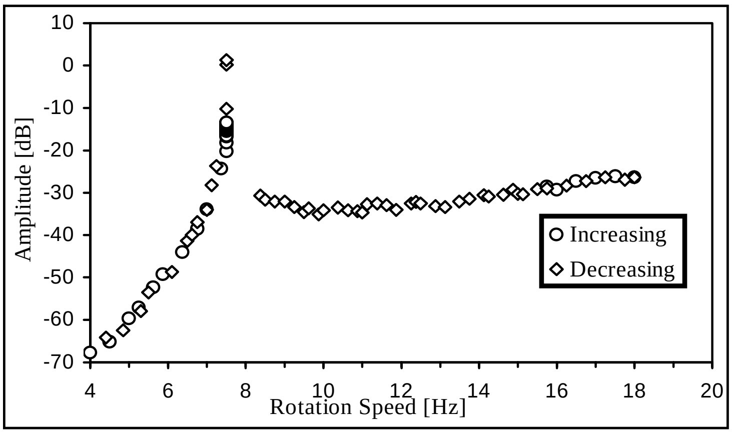

Fig 05 - Amplitude of the first harmonic vs. rotation speed of the motor

In figure 05 we see amplitude of the vibration of the beam against the rotation speed of the motor and we can see the region of frequency in which the response of the system is unstable (7.5 Hz to 8.375 Hz). For increasing voltage, it's not possible to stabilize the system between 7.5 Hz to 15.75 Hz. Figure 06 presents a spectrum of the response of the system near the resonance of the beam. Note that there is clear evidence of second, third, etc... harmonics. However, attempts to activate sub and super-harmonic as well as parametric resonance were not successful (Mook et al, 1997).

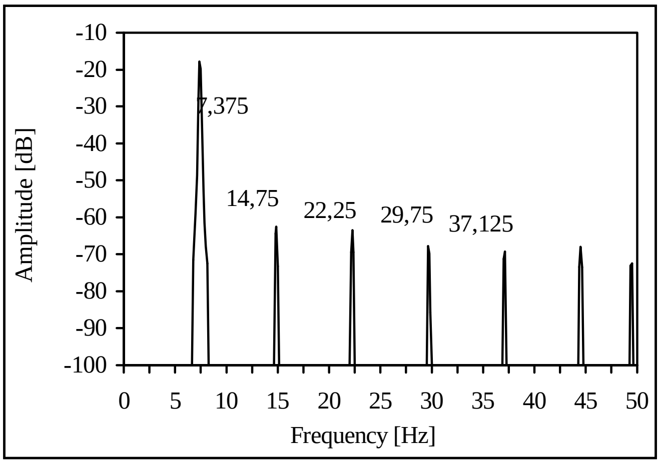

Fig 06 - Spectrum of the response of the system vs. rotation speed of the motor

4. SIMULATION RESULTS

To integrate equations (09)-(11) we used a 5th order varying step Runge-Kutta integrator. The parameters we have used to our analysis are shown in table 01. Note that we have, for instance, neglected the influence of the gravity. However, we have considered three values for the nonlinear stiffness (null, negative and positive). Hence we can infer whether the response observed in the laboratory tests may be due to presence of nonlinear terms like this.

Table 01 - Parameter Values used for simulation

Parameter	Value	Parameter	Value
ξ	8.0000e-2	λ	2.0000e+2
ξ	1.4958e-3	μ	2.8643e-3
σ	-1.0884e-3 | 0 | 1.0884e-3	ρ	1.5915e+0
κ	0	ε	3.235e-1

Figures 07 to 10 show the same kind of data of figures 03 to 06, but for simulated results. In this case we have neglected the influence of nonlinear stiffness and the gravity. In figures 07, 08 and 09, the dotted line refers to the system without interactions between rotational and translational motion. Figure 11 shows a pretty large range of power for which the resonance curve has an unstable branch.

Note that experimental and simulated results are qualitatively the same, unless for the spectrum, due to the presence of harmonics for the experimental data. This indicates that, probably, the linear model for the stiffness is not good to representing the real system. In fact it’s possible that a nonlinear model should represent the beam in which the motor was fixed. Linear model permit us explain the capture of the motor by the resonance of the system, the rotation jump and the unstable region (figure 09) in the right branch of the resonance curve. However, we have still to explain the frequency response of the system (figure 06). Then we have analyzed the case of nonlinear stiffness, whose results are shown in figures 12 and 13.

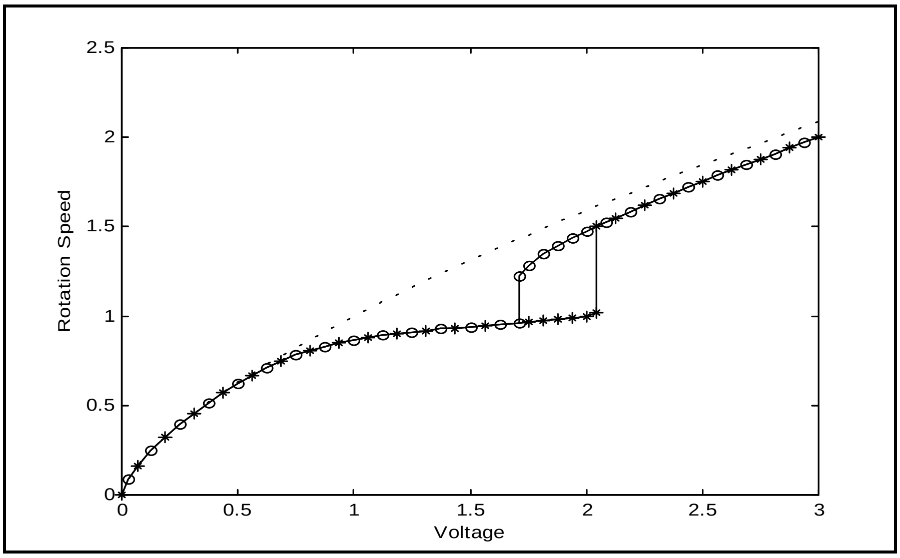

Fig 07 - Rotation speed vs. applied voltage. ∗: *Increasing voltage* o: *Decreasing voltage*

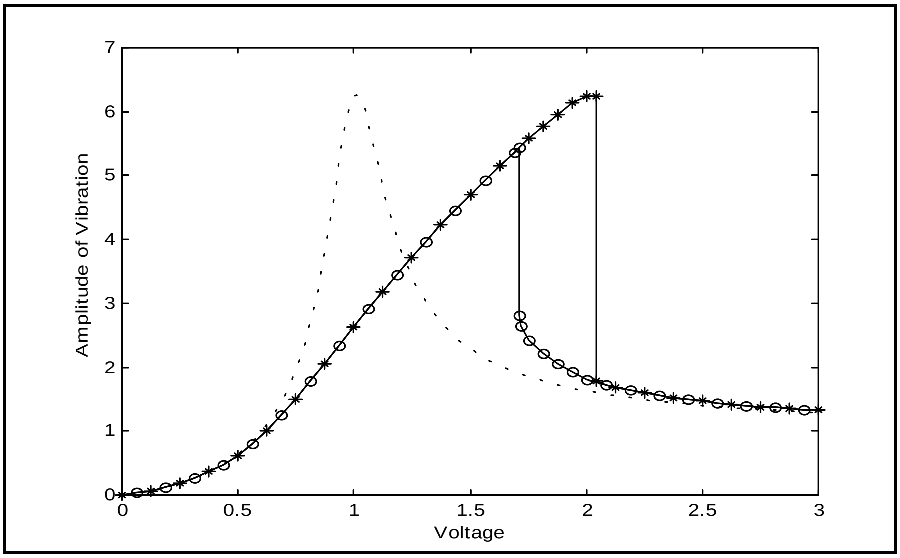

Fig 08 - Vibration Amplitude vs. applied voltage ∗: *Increasing voltage* o: *Decreasing voltage*

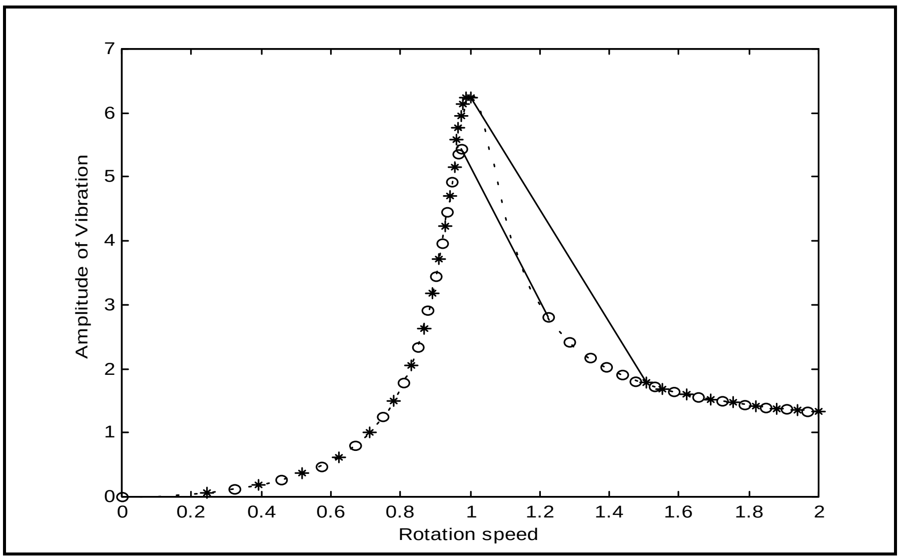

Fig 09 - Vibration amplitude vs. rotation speed. ∗: *Increasing voltage* o: *Decreasing voltage*

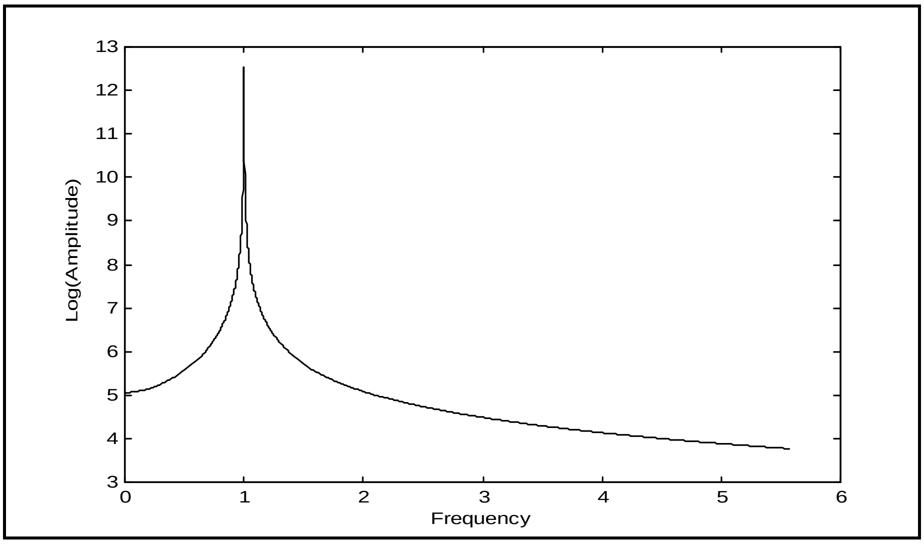

Fig 10 - Spectrum of the response of the beam near the resonance (simulation)

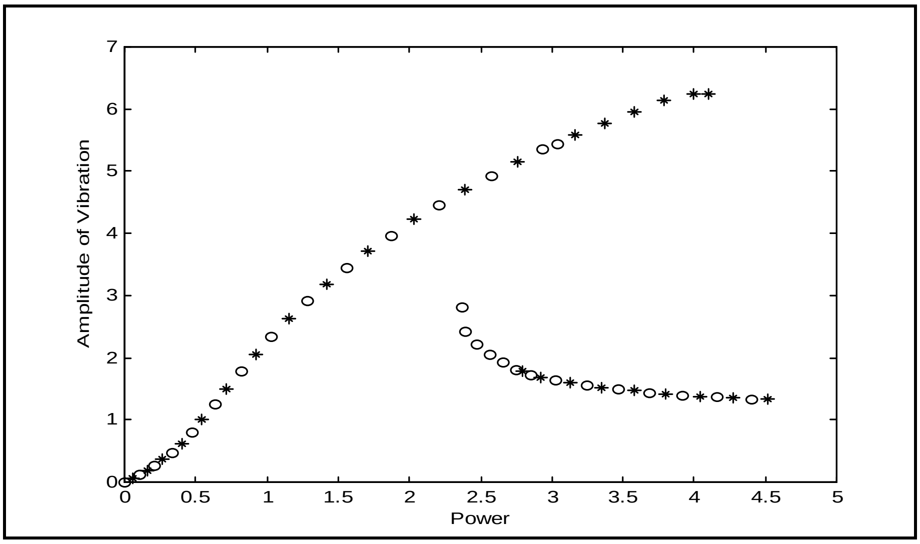

Fig 11 - Vibration amplitude vs. power supplied. ∗: *Increasing power* o: *Decreasing power*

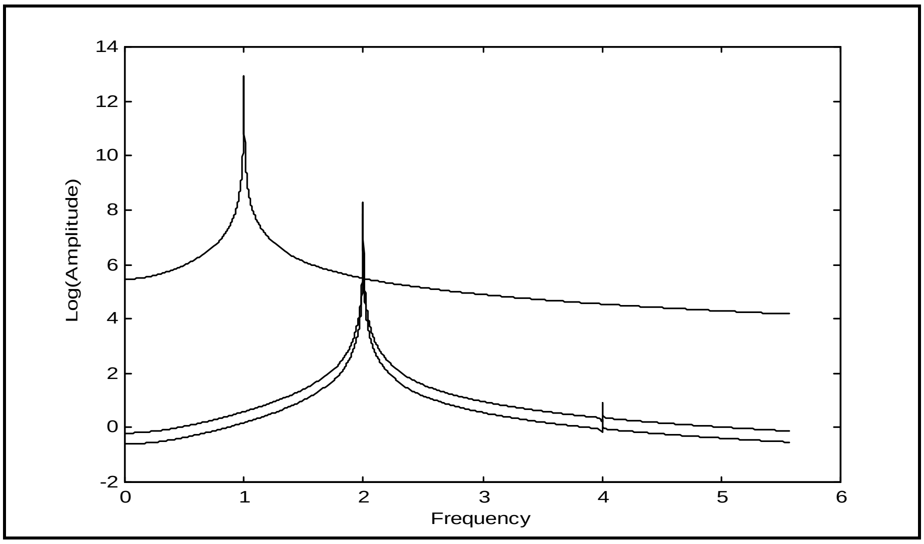

Fig 12 - Spectra for hardening cubic stiffness and null gravity
vibration of the table, rotation of the shaft and demanded current (up to down)

The gravity is equal to zero for the data in figure 12 and non-zero for figure 13. For the rotation speed the data was subtracted of its mean, that is, the results shown consider only the variation of the rotation around its mean value. Then we conclude that the rotation speed is not constant and the frequency of its variation is the same as its mean value. The oscillation frequencies for the current and the displacement of the table are the rotation of the motor and its odd harmonics for null gravity. For non-zero gravity, the spectrum has even and odd harmonics. Then we conclude that the gravity can be responsible by the even harmonics in the spectrum. The results for softening stiffness are the same as that for hardening one. Though the results for linear stiffness combined with gravity is not shown, we know that the even harmonics are present in the spectrum for this case. There are more results to show, involving serial and compounded motors, but there's no space here to present them.

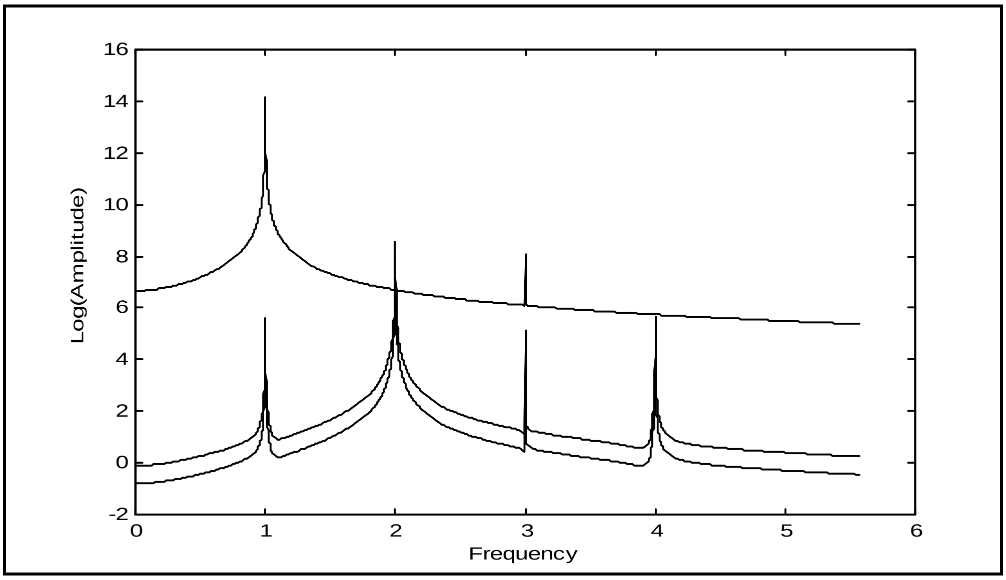

Fig 13 - Spectra for hardening stiffness and non-zero gravity
vibration of the table, rotation of the shaft and demanded current (up to down)

5. CONCLUSION

Some aspects of the interactions between the dynamics of the electric circuit of an armature controlled DC motor and the dynamics of its foundation has been explored in this paper. Instead of considering an experimentally obtained torque-rotation curve to take in account the coupling between the dynamics of the motor and the dynamics of the foundation, we have applied directly the governing equations of the electric circuit of the motor. The advantage is that, now, it’s very easy to change the parameters of the motor in the simulation program, as well as, it’s easy to consider several kinds of motors, even AC motors, whose governing equations are more complex.

Some progress were made. Particularly we cite:

- The interactions between the dynamics of the motor and the dynamics of the base induce a non-constant rotation speed of the motor. The oscillation frequency of the rotation speed is equal to its mean value.
- The gravity can induce even harmonics in the frequency response of the system
- The cubic term for nonlinear stiffness is responsible, as we could expect, for odd harmonics in the frequency response of the system.
- The current demanded by the motor varies with the same frequencies of the vibration of the base. We have not studied yet if there is a problem when this oscillation frequency is equal to

the resonant frequency of the electrical circuit of the motor. We expect that there will be no problem for DC motor, but the same cannot be said for AC motor.

- The new presentation form of the governing equations of the systems permits us to analyze easily the dependence of the inertia terms with the rotation of the motor's shaft. From our point of view, the way to change some control parametrs can be easier visualized.
- The consideration of the motor's governing equations permits us to analyze what happens with the current in the motor's circuit. Consequently, we can describe the torque curve better in comparison with the conventional procedure, which imposes a particular characteristic for the torque-rotation curve. Note that we can also take easily in account the characteristic of load (external) curve.

There are other phenomena that take place in 2-dimensional and 3-dimensional non-ideal problem. A more complete study of the couplings in a 3-dimension system is being developed by both experimental and numerical analysis.

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